GRAVITATIONAL MASS OF MAGNETOSTATIC FIELD

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Abstract. Maxwell's displacement current equation is interpreted in the light of recent work to show that static magnetic field in free space should have a colocated and contemporaneous mass that is neither embodied in, nor can be anticipated from, the mass-energy relation. Thus magnetostatic field in the universe represents an 'invisible' mass. Some consequences are discussed.

1. Introduction

Magnetostatic fields of varying strengths permeate much of the mostly empty space in the universe – from the weak fields in the galactic and intergalactic mediums to the ultrastrong fields in the vicinity of neutron stars. Aspects of the energy in a static magnetic field discussed recently (De, 1993, 1994a) show that this reservoir of energy, even in far locations not in instant communication with the source of the field, can be locally drawn down and partially transformed to other forms such heat added to a fluid (De, 1994b). The heat energy causes the mass of the fluid to increase in accordance with the mass-energy relation (see e.g., Møller, 1966), indicating a source of available mass in the original reservoir. This raises the question, going beyond the mass-energy relation, as to whether magnetostatic field in absence of matter, i.e. in vacuum, does not itself possess a measurable mass. This point is developed here by interpreting Maxwell's displacement current concept. While the resultant mass is much too small to be of interest in terrestrial applications, its place in astrophysical situations merits discussion.

The present paper suggests a colocated and contemporaneous mass of magnetostatic field, and not an *equivalent* mass in the sense that the field energy can be obtained by consuming mass. The mass-energy relation *does not* assign a colocated and contemporaneous mass to magnetostatic energy in empty space. It will be shown further that the result obtained in the present paper cannot even be anticipated from this relation. In the Electromagnetic (EM) Theory, although it has long been noted that there is an inertial character associated with EM fields (see, e.g., Stratton, 1941), there exists no specific suggestion that a static magnetic field in vacuum has a colocated mass.

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2. Definition of Inertial Mass in Empty Space

The underlying idea of an actual and observable mass residing in perfect vacuum does not appear to have been put forth before. It may be noted though that even in the mass-energy relation the mass depends only on the amount of energy, and not on any material property (e.g., mass, density, composition) of the body to which the energy is added. This concept already advances a long way towards that of a mass without material content. It is necessary, however, to adopt a legitimate definition of mass that can be applied to empty space. It is most convenient to begin the discussion by considering inertial mass, and then to generalize the discussion to gravitational mass.

A workable definition of inertial mass that can be adopted to empty space happens also to be in one view the most unambiguous definition of mass in physics: An inertial mass possesses simultaneously a measurable momentum and a measurable velocity that is less than the velocity of light (Kompaneyets, 1965). The quantitative definition is that the inertial mass equals the scalar ratio of the momentum to the velocity. Thus, to measure a mass one must consider it to be in motion.

The above definition is consistent with the definition of mass as the scalar ratio of force to acceleration. However, the momentum-based definition can blend more easily with the relevant concepts of classical EM Theory.

3. Measurement of Inertial Mass in Empty Space

It is next necessary to specify a way to measure the momentum contained in EM fields in empty space. This can be done by employing a device termed a force-measuring antenna (De, 1993).

A force-measuring antenna (FMA) is quite simply an electric field sensing antenna or conductor mounted on a force transducer that senses any mechanical force on the antenna. Beyond serving as a measuring implement, this device provides a crucial conceptual foundation for the present discussion: If the transducer senses a mechanical momentum, then the momentum carried by the incident EM fields is by definition a mechanical momentum. By establishing this fact, the device permits one to go beyond a certain long-standing paradox (see Section 5) of the EM Theory, and explore areas that have not been traditionally explored.

4. Definition of Reference Frames

The following discussion makes use of a certain magnetic force on a pure dielectric (Brevik, 1976) that has occupied an obscure place in the development of the EM Theory. It has been suggested that the study of this force can help complete certain incompletenesses in this development (De, 1988, 1993). Initially, a unit cube of a lossless, linear dielectric material of mass ρ and polarizability χ is considered. It

is accelerating under an external force across a uniform static magnetic field $\mathbf{B}_0$ in vacuum, with one edge of the cube parallel to the field and one face perpendicular to the direction of motion. The source of this magnetic field defines the rest frame of the discussion. The cube has a velocity $\mathbf{v}$ ($v \ll c$, the speed of light) and an acceleration $\mathbf{f}$, and constitutes the moving frame.

Two experiments will now be considered: An experiment in the moving frame by Observer A stationed on the dielectric cube platform, and one in the rest frame by Observer B when the dielectric is replaced by an infinitely conducting material.

5. Measurement in the Moving Frame

In the moving frame there is an electric field $\mathbf{E} = \mathbf{v} \times \mathbf{B}_0$. If $\mathbf{B}$ is the induced magnetic field ($B \ll B_0$ for simplicity) in the same frame, then $\mathbf{E}$ and $\mathbf{B}$ satisfy Maxwell's displacement current equation:

$$\nabla \times \mathbf{B}/\mu_0 = \chi\varepsilon_0\dot{\mathbf{E}} + \varepsilon_0\dot{\mathbf{E}} \tag{1}$$

where μ_0 and ε_0 are the magnetic permeability and the dielectric permittivity of vacuum. The time-variation arises during acceleration. It is now noted that the vacuum displacement current, the second term on the right hand side, was originally predicted by Maxwell by extending the material current (the first term) to vacuum, and was later experimentally verified. Taking the cross product of the above equation with $\mathbf{B}_0$ one obtains

$$(\nabla \times \mathbf{B}) \times \mathbf{B}_0/\mu_0 = \chi\varepsilon_0\dot{\mathbf{E}}_0 \times \mathbf{B}_0 + \varepsilon_0\dot{\mathbf{E}} \times \mathbf{B}_0. \tag{2}$$

The first term on the right hand side is a theoretically predicted and experimentally verified mechanical force on a dielectric (Brevik, 1976). By invoking Maxwell's reasoning (i.e., by extending the dielectric force to vacuum), the presence of a vacuum force term could be predicted. However, such a term has followed naturally from the vacuum current, and can be written as the time-rate of change of a momentum density

$$\mathbf{G}_0 = \varepsilon_0\mathbf{E} \times \mathbf{B}_0. \tag{3}$$

This quantity has long been described as an abstract *electromagnetic* momentum to distinguish it from the mechanical momentum contained in the first term on the right hand side of Equation (2). However, as mentioned before, $\mathbf{G}_0$ has been shown to be observable as a time-varying mechanical momentum using an FMA. If $\mathbf{G}_0$ can be observed as a mechanical momentum, then it is by definition a mechanical momentum. In this way the present discussion can circumvent the unresolved historical controversy in the EM Theory regarding what is electromagnetic momentum and what is mechanical momentum (*op. cit.*).

Observer A in the moving frame can measure his acceleration $\mathbf{f}$, and the time derivative of $\mathbf{G}_0$. By making these measurements for several values of $\mathbf{f}$, he will find (e.g., by making a graphical plot of the observations) that

$$\dot{\mathbf{G}}_0 = -C_1\mathbf{f} \tag{4}$$

which yields a numerical value for the constant C_1. He can now integrate the above equation to obtain

$$\mathbf{G}_0 = -C_1\mathbf{v} + \mathbf{C}_2. \tag{5}$$

6. The Meaning of C_1 and $\mathbf{C}_2$

The vector integration constant $\mathbf{C}_2$ is now interpreted to be the value of $\mathbf{G}_0$ when the observer is at rest, and can be set equal to zero as a choice. This leads A to conclude that the momentum flow $\mathbf{G}_0$ is due entirely to the velocity of his frame, or that the velocity $\mathbf{v}_0$ and the acceleration $\mathbf{f}_0$ of the momentum flow in his frame are equal to $-\mathbf{v}$ and $-\mathbf{f}$, respectively. Hence

$$\mathbf{G}_0 = C_1\mathbf{v}_0 \tag{6}$$

which is the unambiguous definition in physics of an inertial mass C_1. This quantity is therefore an intrinsic mass residing in vacuum, and owes its presence to the magnetic field $\mathbf{B}_0$. Upon combining Equation (3) with the above equation, one finds

$$C_1 = \varepsilon_0 B_0^2. \tag{7}$$

7. Magnetic Field and Motion

The experiment described above is somewhat analogous to the following situation: An observer W accelerates through sill air. He therefore feels a gust of wind. By measuring the ram pressure of the wind on a plate held *perpendicular* to the direction of motion, W can determine the momentum of the wind and hence the mass density of air.

Returning to Observer A, one can now consider motion in arbitrary directions with respect to the magnetic field. By orienting the FMA for peak signal, this observer can determine the direction of the momentum flow. Since he knows the direction of the magnetic field, he can make allowance for the angle between the two directions in Equation (4). He will then measure the same value of the mass density C_1 regardless of the direction of travel. This shows that the mass of magnetostatic field is independent of the directional nature of magnetic field.

The singular exception to this arises when the motion is exactly parallel to the magnetic field direction. Here the momentum $\mathbf{G}_0$ and the velocity $\mathbf{v}_0$ of the momentum flow are both zero, so that $C_1 = 0/0$ becomes indeterminate. Such indeterminacy related to motion parallel to a magnetic field has been considered in

traditional EM Theory as a paradox, indicating the hazard of assigning motion (or not) to the magnetic field. However, the singularity here is not an issue of the EM Theory in particular, but of measurement science in general.

First, the case of Observer W performing a non-EM measurement can be considered. If, instead of holding his plate perpendicular (i.e. at 90 degrees) to the direction of travel, he holds it at an arbitrary angle and makes an allowance for it, he always measures the same mass density of air regardless of the angle. However, the mass becomes indeterminate if the angle is exactly 0 degree. This does not disprove that air has a mass. It simply means that for this particular configuration, the measurement technique fails. Many such instances of non-EM measurements can be described.

Second, one can consider the instance of Observer A performing an EM measurement different from the one described. He makes a noncontroversial measurement of the magnetostatic *energy density* which is an established quantity of physics. He measures the magnitude and the direction of the motional electric field $\mathbf{E}$ using conventional instruments, calculates the magnetic field upon taking into account the angle between this $\mathbf{E}$ and the known velocity $\mathbf{v}$, and then reports the energy density $U_0 = B_0^2/2\mu_0$. He measures the same value of U_0 for every direction of motion except the singular case when he travels exactly parallel to the magnetic field, where the energy density becomes indeterminate. This case does not render the energy density meaningless, nor says that is it nonexistent, nor assigns any directionality to it.

There is nothing inexplicable or paradoxical about the above instances. The observer's remedy in all such instances is to slightly perturb the angle in question if he encounters the indeterminacy. The same comment applies also to the present discussion.

8. Measurement in the Rest Frame

Next, the viewpoint of Observer B is considered. The dielectric cube is now replaced by a cube of infinitely conducting material. The cube is then assumed to be attached to the rest frame by a spring having a spring constant k. It is an 'ideal' spring in that it is massless, and electrically and magnetically inert. The instantaneous position of the cube is $\mathbf{x}$, and the magnetic field $\mathbf{B}_0$ is perpendicular to the x axis. In absence of the magnetic field, the time period of mechanical vibration of the system is

$$T = 2\pi\sqrt{\rho/k}. \tag{8}$$

Knowing k and measuring T, B can find the inertial mass ρ.

When the magnetic field is present, the equation of motion of the cube becomes

$$\rho\ddot{\mathbf{x}} = -k\mathbf{x} - \varepsilon_0\dot{\mathbf{E}} \times \mathbf{B}_0 \tag{9}$$

which can be solved to find that the time period now is

$$T_B = 2\pi\sqrt{(\rho + \rho_0)/k} \tag{10}$$

with

$$\rho_0 = \varepsilon_0 B_0^2. \tag{11}$$

Upon measuring T_B and knowing k, B finds that the inertial mass of the cube is $\rho + \rho_0$. Since the experiment utilizes a conservative (lossless) medium, the slowing of the time period cannot be ascribed to any losses. Since the increase in the mass of the cube is independent of the original mass, the measured value of ρ_0 remains unchanged as ρ is made arbitrarily small. Thus the above experiment amounts to a clear and unambiguous measurement of the mass density of the magnetic field. The same comments about the directionality of the magnetic field apply to the above discussion as in Section 7.

Thus Observers A and B, in different frames of reference and pursuing different measurement methods, both come to the same absolute conclusion: Magnetostatic field in vacuum has a colocated and contemporaneous mass in accordance with the established definition of mass in physics. It follows that

$$\rho_0 \equiv C_1. \tag{12}$$

9. Generalization to Gravitational Mass

The discussion so far has dealt with inertial mass. Since the inertial mass and the gravitational mass are one and the same (Cf. Møller, 1966), the mass in Equation (12) can be treated as gravitational mass. It is desirable, however, to have an independent proof to support this identification in the case of the newly introduced mass that resides in empty space. To do this, one can consider a physical circumstance where both the inertial mass and the gravitational mass are simultaneously manifest.

This is indeed the case with a simple pendulum. The cube of Section 8 is now assumed to be the bob of this pendulum, with B_0 being the Earth's magnetic field, assumed horizontal. The motion of the bob is horizontal, and perpendicular to the magnetic field. If ρ_G is the gravitational mass of the bob, g the acceleration due to gravity, and l the length of the pendulum, then the equation of motion of the pendulum is

$$\rho l\ddot{\theta} = -\rho_G g\theta - \varepsilon_0 l\ddot{\theta} B_0^2 \tag{13}$$

where θ is the angular displacement of the pendulum, assumed small. Since the time-period of the pendulum cannot depend of the gravitational mass of the bob, it follows that

$$\rho_G = \rho + \rho_0, \tag{14}$$

confirming that ρ_0 is a gravitational mass residing in empty space.

10. ρ_0 and the Mass-Energy Relation

Attempts to connect the result of this paper to the mass-energy relation face the following difficulty: This relation applies only to energy *that is added to a material medium.* The mass-energy relation does not assign a colocated mass to pure magnetostatic energy in vacuum. Thus there is no basis for a direct intercomparison between this relation and the result of the present paper. This also mean that the result is not in conflict with the mass-energy relation.

If one were to conjecturally extend the mass-energy relation to vacuum without regard to the unwarranted nature of this procedure, one would equate the magnetic energy density $U_0 = B_0^2/2\mu_0$ to the quantity $\rho_{me}c^2$, where ρ_{me} is the mass density anticipated from the relation. This results in

$$\rho_{me} = \varepsilon_0 B_0^2/2 \tag{15}$$

which differs from C_1 or ρ_0 by a factor 2. This shows that the result of Equation (7) or (11) cannot even be anticipated from the mass-energy relation. One may wish, for whatever reason, to preserve the mass-energy relation even in vacuum by interpreting the factor 2 as a discrepancy between $U_0 = B_0^2/2\mu_0$ and a seemingly natural alternative energy density, $U_0 = B_0^2/\mu_0$. This duality, however, *would* lead to legitimate paradoxes and violations. The conclusion then is that the result of the present paper is neither embodied nor contemplated in the mass-energy relation. It may be that mass, and not energy, is the true attribute of a static magnetic field.

When magnetostatic energy is drawn down and partially transformed to heat energy of a fluid (De, 1994b) with resultant increase in the mass of the fluid, the mass-energy relation does of course apply to the added *heat* energy and the increase in mass. However, this does not lead to Equation (15).

The general result of the present paper can be experimentally verified by weighing a superconducting coil of inductance L with and without a steady current I flowing through it. According to the present paper, the mass of the coil should increase by a quantity Δm when energized, where

$$\Delta m = \int \rho_0 \, dV = LI^2/c^2, \tag{16}$$

and where the volume integral extends substantially over the region of the magnetic field in the empty space next to the coil. It may be noted that this test does not require the magnetic field or the observer to be in motion. A verification of the above equation amounts to a verification of the following conclusions:

(i) Magnetostatic field in free space has an actual mass;

(ii) There exists in the universe a mass without material content; and

(iii) These results are not contained in the mass-energy relation.

11. Applicability Considerations

Since in general the mass of magnetostatic field is very small, it is best compared with the lightest familiar component of the material mass in the universe, namely, electrons. It is convenient to express the mass density of magnetostatic field in terms of the equivalent number of electrons per cubic centimeter:

$$N_e \sim 10^5\, B_{og}^2 \tag{17}$$

where the magnetic field now is in units of gauss. This relation permits one to compare N_e with the electron number density $\mathrm{N_e}$ in various astrophysical situations. It may be noted that while in local concentrations of magnetic field such as a sunspot the ratio $N_e/\mathrm{N_e}$ can far exceed unity, for the mean galactic medium the ratio is quite small ($\sim 10^{-4}$). Thus, while magnetostatic field in the universe does represent an invisible mass, it is much too weak to account for the so-called cosmological dark matter (Saunders *et al.*, 1991).

Magnetic fields in excess of 10^{12} G are known to exist in the largely empty space near neutron stars where exotic physical processes are thought to be operative (Harding, 1991; Beskin and Gurevich, 1993). Here the values of N_e exceed 10^{29} cm^{-3}. The mass density ρ_0 far exceeds that due to the disperse subatomic particles in the region. This fact needs to be incorporated in the physics of neutron star atmospheres. One might consider if the magnetic field is not so intense in part because the field has been compressed by the gravity of the star. This suggested process is distinct from the magnetohydrodynamic compression of magnetic field when a body of conducting gas gravitationally collapses.

12. Remarks

Many questions will no doubt arise in attempting to reconcile the mass ρ_0 with the known properties of magnetostatic fields. For example, a magnetic field is dependent on the frame of reference in which it is measured so that ρ_0 will change accordingly. This means that in a distended region of magnetostatic field the mass will appear redistributed in space when the observer shifts from frame to frame. Second, if the magnetic field is caused to vary faster and faster with time so that the magnetostatic energy gradually assumes the form of EM radiation, then ρ_0 must gradually vanish. This is because photons which constitute the radiation have no rest mass. This implies a functional relationship between ρ_0 and the period τ of time variation, $\rho_0(\tau)$, that may be worth exploring in the context of the study of the essential nature of vacuum itself (Boyer, 1985; Podolnyi, 1986; Puthoff, 1989). Third, one may wonder as to the mechanism by which a distended region of magnetic field (devoid of matter, say) in the universe might gravitationally attract a material mass (e.g., a galaxy). This question may be answerable once the mechanism of gravitation between two material masses is understood. In this context, the

problem of gravitational support of a planetary dipole magnetic field also needs to be addressed. Finally, Equation (14) represents a rudimentary connection between magnetism and gravitation, and as such may be of interest in the search for a theory unifying Electromagnetism and Gravitation.

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